CHAPTER 11

SERIES AND PARALLEL CIRCUITS

11.1 SINGLE RESISTOR CIRCUITS

A circuit may contain one or more resistor. The multi-resistor circuits can have their resistors connected together in many different ways but the application of a standard set of problem solving rules will always simplify the circuit to a simple circuit consisting of an equivalent resistor and an e.m.f. source. A circuit consisting of a single resistor and an e.m.f. source is shown in figure 11.1.

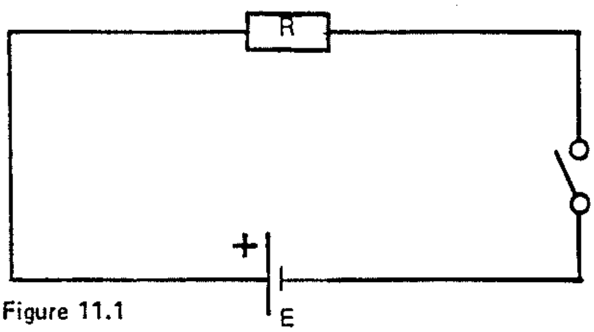


Figure 11.1

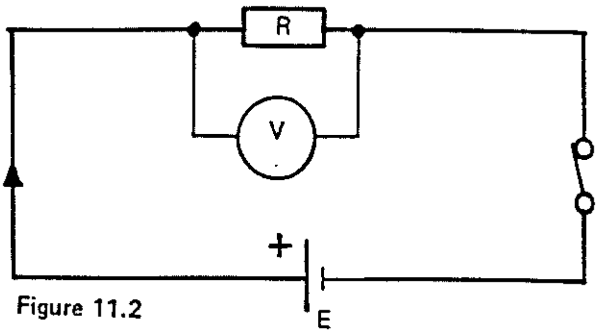


Figure 11.2

When the switch is closed conventional current flows in the external circuit, in the direction indicated in Figure 11.2.

Only one value of current exists in the circuit, all electrons leaving the source, pass through the resistor to the opposite side of the source.

All the Ohm's Law equations apply to the circuit.

$$E = IR$$

$$I = \frac{E}{R}$$

$$R = \frac{E}{I}$$

When the current flows through the resistor a voltage (IR) is produced across the resistor. This internal voltage is known as the 'potential difference' across the resistor and is identified by the capital letter 'V', whereas a source voltage is indicated by the letter 'E'.

11.2 SERIES CIRCUITS

Resistors are connected in series when they provide a single path for the passage of electrical current. (Figure 11.3).

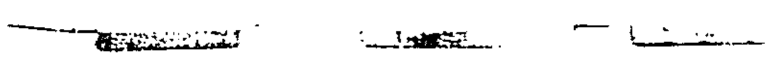


Figure 11.3

Electrons entering from either end of the resistor grouping must pass through each and all of the resistors. A series circuit containing two resistors and a source is shown in figure 11.4.

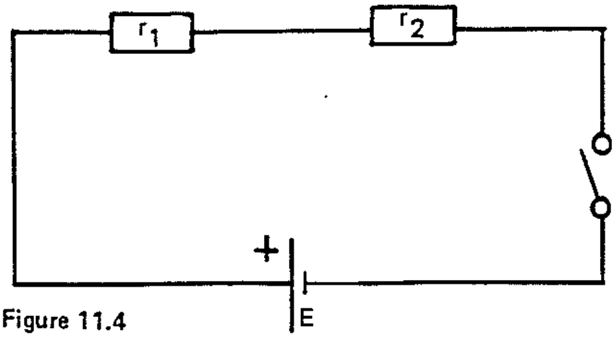


Figure 11.4

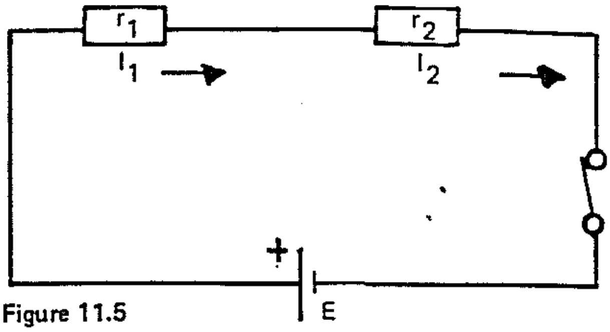


Figure 11.5

When the switch is closed, (figure 11.5) current moving from the positive side of the source through the circuit meets opposition first from r_1, then from r_2. as it progresses to the negative terminal of the source. The total opposition to current movement is r_1 plus r_2, or the total resistance (R) to current movement is -

$$R = r_1 + r_2 \text{ ohms}$$

As there is only one path in the circuit there will only be one current . (Figure 11.4).

$$I = I_1 = I_2$$

This current is calculated by using the Ohm's Law equation -

$$I = \frac{E}{R}$$

Where - E = applied e.m.f.

R = total circuit resistance

From Ohm's Law, if current flows in a resistor a voltage appears across the resistor. The value of this voltage is equal to the product of the current in the resistor and resistance of that resistor through which this current is flowing. Thus current flow in this series circuit produces two internal voltages. (Figure 11.6).

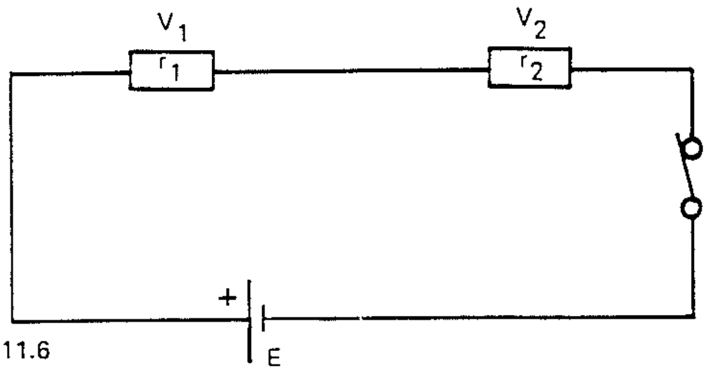


Figure 11.6

The applied e.m.f. is equal to the sum of the individual voltages across the resistors.

$$E = V_1 + V_2$$

Where - $V_1 = Ir_1$ and

$V_2 = Ir_2$

Example 11.1

A 10 ohm and a 20 ohm resistor are connected in series to a 120 volt d.c. supply. Calculate —

(a) the total resistance of the circuit
(b) the current in the circuit
(c) the potential differences across each resistor.

$E = 120\ V$

$r_1 = 10\ \Omega$

$r_2 = 20\ \Omega$

$R = ?\ \Omega$

$I = ?\ A$

$V_1 = ?\ V$

$V_2 = ?V$

$R = r_1 + r_2$

$= 10 + 20$

$= 30$ ohms

$I = \frac{E}{R}$

$= \frac{120}{30}$

$= 4A$

$V_1 = Ir_1$

$= 4 \times 10$

$= 40\ V$

$V_2 = Ir_2$

$= 4 \times 20$

$= 80\ V$

Note: That $V_1 + V_2 = E$

$40\ V + 80\ V = 120\ V.$

Example 11.2

Three resistors r_1, r_2 and r_3 of 4 Ω, 6 Ω and 10 Ω respectively are connected in series to a 200 volt d.c. supply. Calculate the:-

(a) total resistance of the circuit

(b) current drawn from the supply

(c) voltage drop across each resistor

$E = 200V$

$r_1 = 4\ \Omega$

$r_2 = 6\ \Omega$

$r_3 = 10\ \Omega$

$R = ?\ \Omega$

$I = ?A$

$V_1 = ?V$

$V_2 = ?V$

$V_3 = ?V$

(a) $R = r_1 + r_2 + r_3$

$= 4 + 6 + 10$

$= 20\ \Omega$

(b) $I = \frac{E}{I}$

$= \frac{200}{20}$

$= 10A$

(c) $V_1 = Ir_1$

$= 10 \times 4$

$= 40V$

$V_2 = Ir_2$

$= 10 \times 6$

$= 60V$

$V_3 = Ir_3$

$= 10 \times 10$

$= 100V$

Check - $E = V_1 + V_2 + V_3$

$200 = 40 + 60 + 100$

$= 200V$

11.3 PARALLEL CIRCUITS

Resistors are said to be connected in parallel when they provide more than one path for the passage of electrical current. (Figure 11.7).

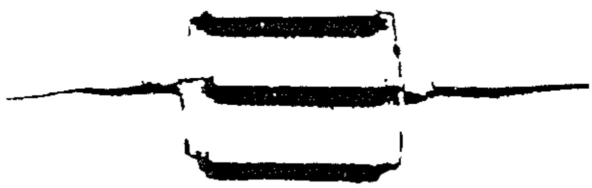


Fig 11.7

Electrons entering from either end of the resistor grouping may pass through any of the resistors. A parallel circuit consisting of two resistors is shown in figure 11.8.

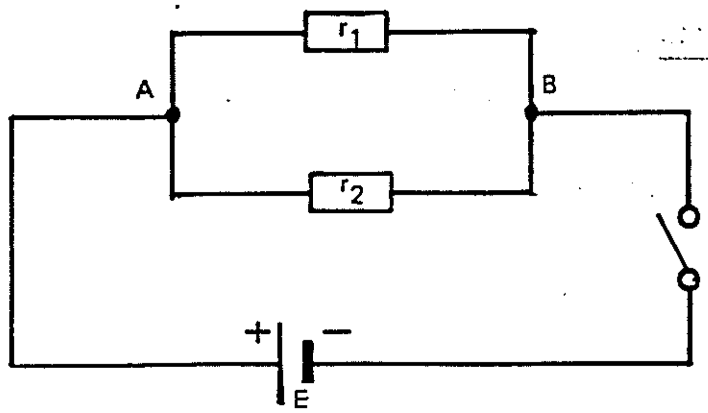
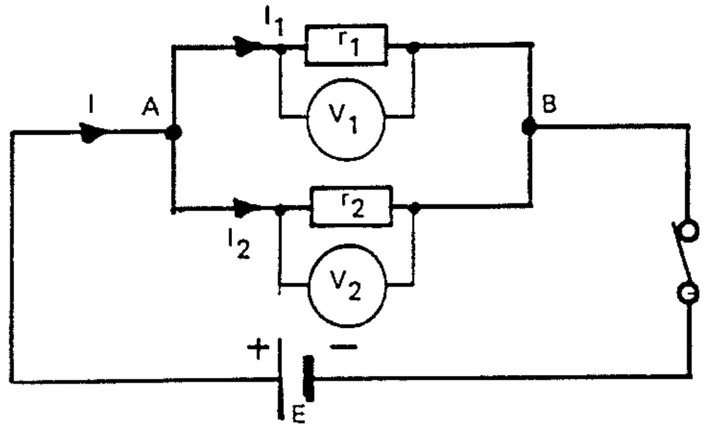


Figure 11.8

When the switch is closed, current leaving the source and reaching position A, will have two paths available through which it may travel. On reaching point B the current in the two paths combine to give the same total value as was flowing in the circuit prior to point A.



Neglecting wire resistance the potential at the points A and B will be the same as that of the source making V_1 and V_2 equal to the source voltage E. (Figure 11.9).

$$E = V_1 = V_2$$

The total current in the circuit is the sum of the individual branch current.

The total current in the circuit is given by —

$$I = I_1 + I_2$$

or $$\frac{E}{R} = \frac{V_1}{r_1} + \frac{V_2}{r_2}$$

but $$E = V_1 = V_2$$

substituting E for V

$$\frac{E}{R} = \frac{E}{r_1} + \frac{E}{r_2}$$

(after dividing both sides by E)

$$\left(\frac{1}{R}\right) = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

and transposing make the total resistance —

$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$$

Example 11.3

A resistor of 3 ohms is connected in parallel to a resistor of 6 ohms. The complete circuit is then supplied from a 90 volt d.c. source. Calculate -

(a) the total resistance of the circuit
(b) the current in each resistor
(c) the total current drawn from the supply

$E = 90\ V$

$r_1 = 3\ \Omega$

$r_2 = 6\ \Omega$

$R = ?\ \Omega$

$I_1 = ?\ A$

$I_2 = ?\ A$

$I = ?\ A$

(a)
$$R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{1}{\frac{1}{3} + \frac{1}{6}} = \frac{1}{\frac{2+1}{6}} = \frac{1}{\frac{3}{6}} = 2\ \Omega$$

(b)
$$I_1 = \frac{E}{r_1} = \frac{90}{3} = 30\ A$$

$$I_2 = \frac{E}{r_2} = \frac{90}{6} = 15\ A.$$

(c)
$$I = I_1 + I_2 = 30 + 15 = 45\ A$$

Check -
$$I = \frac{E}{R} = \frac{90}{2} = 45A$$

Example 11.4

Three resistors r_1, r_2 and r_3 having resistances of 2 Ω, 3 Ω and 6 Ω respectively are connected to a 30 volt supply. Calculate the:-

(a) total resistance

(b) current drawn from the supply

(c) current in each resistor.

$E = 30V$

$r_1 = 2\ \Omega$

$r_3 = 3\ \Omega$

$r_3 = 6\ \Omega$

$R = ?\ \Omega$

$I = ?A$

$I_1 = ?A$

$I_2 = ?A$

$I_3 = ?A$

(a) $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$= \frac{3 + 2 + 1}{6}$

$= \frac{6}{6}$

$= 1$

$R = 1\ \Omega$

(b) $I = \frac{E}{R}$

$= \frac{30}{1}$

$= 30A$

(c) $I_1 = \frac{V_1}{r_1}$

$= \frac{30}{2}$

$= 15A$

$I_2 = \frac{V_2}{r_2}$

$= \frac{30}{3}$

$= 10A$

$I_3 = \frac{V_3}{r_3}$

$= \frac{30}{6}$

$= 5A$

Check $I = I_1 + I_2 + I_3$

$30 = 15 + 10 + 5$

$= 30A$

Equations in this chapter

In a series circuit –

$$R = r_1 + r_2$$

$$I = I_1 = I_2$$

$$E = V_1 + V_2$$

In a parallel circuit –

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} \quad \text{or} \quad R = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$I = I_1 + I_2$$

$$E = V_1 = V_2$$

TUTORIAL 1.11

(1) Three resistors of 2 Ω, 5 Ω and 6 Ω are connected in series to a 130V d.c. source. Calculate:-

(a) the total resistance of the circuit

(b) the current drawn from the source

(c) the potential difference across each resistor.

(2) A d.c. supply of 220V is applied to a circuit consisting of a 220 Ω, a 180 Ω and a 40 Ω connected in series. Calculate -

(a) the total resistance of the circuit

(b) the current drawn from the d.c. supply

(c) the potential difference across each resistor.

(3) Three resistors of 5 Ω, 10 Ω and 15 Ω are connected in parallel to a 120V d.c. supply. Calculate -

(a) the resistance of the circuit

(b) the current drawn from the supply

(c) the voltage across the 5 Ω resistor

(d) the current through the 15 Ω resistor

(4) Three resistors, r_1, r_2 and r_3 having values of 6 Ω, 9 Ω and 18 Ω respectively are connected in parallel to a 108 volt source. Calculate -

(a) the total resistance of the circuit

(b) the current drawn from the source

(c) the potential difference across the 18 Ω resistor

(d) the current in the 6 Ω resistor.

(5) A parallel circuit consists of three resistors of 8 Ω, 12 Ω and 24 Ω connected to a 120 volt supply. Calculate -

(a) the total resistance of the group

(b) the current drawn from the supply

(c) the current in the 12 Ω resistor

(d) the potential difference across the 18 Ω resistor.